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NEW YORK UNIVERSITY

Institute of Mathematical Sciences

Division of Electromagnetic Research

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Propagation of Electromagnetic Pulses Around the Earth

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Abstract

The propagation of electromagnetic pulses around the earth is investigated analytically. The pulses are assumed to be produced by a vertical electric or magnetic dipole. The earth is treated as a homogeneous sphere of either finite or infinite conductivity and the atmosphere is assumed to be homogeneous. It is found that very short pulses become longer the further they propagate, in addition to diminishing in amplitude. The duration of a pulse which is initially a delta-function increases as Θ^3 , where Θ is the angle between source and receiver. The results are represented as products of several factors, which we call the amplitude factor, the pulse-shape factor, the time-dependent height-gain factor for the source and receiver, and the conductivity factor. Graphs of these factors and of the pulse shape for several cases are given.

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1. Introduction

Suppose an electromagnetic pulse of given shape and amplitude is emitted by a source on or near the ground. We wish to find its shape, amplitude and arrival time at any point beyond the horizon on or near the ground. In particular we wish to determine the effects of the ground conductivity along the propagation path. We will assume that the source is a vertical dipole of either electric or magnetic type and that the earth is a homogeneous sphere. By a slight modification of our procedure, we could also treat the case in which the ground conductivity varies along the propagation path.

The method of solution consists of two steps. First we must obtain the Hertz vector which represents the field due to a time harmonic or periodic source. Then by Fourier superposition of these periodic fields we can obtain the Hertz vector for a delta-function source. Other pulse-type sources can be treated by superposition of delta-function fields.

Since the field due to a periodic source has been determined by numerous authors, our first step is simple--we need merely copy the Hertz vector of this field. This is done in Section 2, the books of V.A. Fock^[1] and H. Bremmer^[2] being used as sources. In Section 3 we write the Fourier integral representing the Hertz vector of the field due to a delta-function source. We then evaluate this integral by the saddle point method and obtain a simple formula as the result for each type of dipole. In Section 4 some graphs based on these formulas and a discussion of the results are given.

We wish to acknowledge our indebtedness to Prof. Bernard Friedman who previously analyzed the field due to a pulsed line source near a perfectly conducting cylinder.

2. The field of a periodic source

Suppose a vertical electric dipole is located at the point $r = \rho$, $\theta = 0$ of a polar coordinate system with the origin at the center of the earth. Then its field can be expressed in terms of a Hertz vector having only a radial component $rU_e(r, \theta, t)$, by the formulas

$$(1) \quad E = \nabla \times \nabla \times (\vec{r}U_e)$$

$$H = \left(\epsilon \frac{\partial}{\partial t} + \sigma \right) \nabla \times (\vec{r}U_e).$$

Similarly, the field of a vertical magnetic dipole at $r = \rho$, $\theta = 0$ can be expressed in terms of a radial Hertz vector $rU_m(r, \theta, t)$ by the formulas

The numbers τ_s^∞ and τ_s^0 are complex, lying on the ray $\arg \tau = \pi/3$. The τ 's of smallest absolute value are

$$(16) \quad \tau_0^\infty = 0.808 e^{i\pi/3}$$

$$(17) \quad \tau_0^0 = 1.856 e^{i\pi/3}.$$

For a highly conducting earth σ_1 is large so δ_e is large and δ_m is small. Therefore the series (14) is useful for computing τ_s in the electric case and (15) is useful in the magnetic case. For the same reason (8) is more convenient for computing f in the electric case, and (9) is in the magnetic case.

3. The field of a delta-function source

The Hertz potential $U(r, \theta, t)$ of the field due to a vertical electric or magnetic dipole having any time dependence can be obtained by Fourier superposition of the periodic potentials $u(r, \theta)e^{-i\omega t}$ described above. Since $k = \omega/c$, where $1/c = \sqrt{\epsilon\mu}$, we can integrate with respect to k instead of ω and obtain

$$(18) \quad U(r, \theta, t) = \frac{c}{2\pi} \int_{-\infty}^{\infty} A(k) u(r, \theta) e^{-ikct} dk.$$

In (18) the Fourier amplitude $A(k)$ is determined by the time dependence and amplitude of the source. With the function $u(r, \theta)$ defined as in the preceding section, a delta-function source has the amplitude $A(k) \equiv 1$. In the absence of the earth, the Hertz potential of the field due to such a dipole would be

$$(19) \quad U_0 = \frac{\delta(t - \frac{R}{c})}{4\pi R}.$$

Here $R = (r^2 + \rho^2 - 2\rho r \cos \theta)^{1/2}$ is the distance from the source to the receiver. Thus with $A(k) \equiv 1$, (18) will yield the diffracted pulse due to the incident pulse (19).

To evaluate (18) we introduce the time T defined by

$$(20) \quad T = t - \frac{\rho\theta}{c} + \frac{a}{c} \cos^{-1}\left(\frac{a}{\rho}\right) + \frac{a}{c} \cos^{-1}\left(\frac{a}{r}\right).$$

This time T is just the time measured from the arrival of the diffracted wave front at the point (r, θ) , assuming that the source starts at $t = 0$. We now insert (8)

or (9) into (18) and set $A(k) = 1$.

Let us first consider the case of a perfectly conducting earth. In this case τ is independent of k and δ is either infinity or zero. Then for $T < 0$ the contour in (18) can be closed in the upper half-plane to yield the value zero for U . This is, of course, to be expected from the definition of T . For $T > 0$ we evaluate (18) by the saddle point method. In the electric case we use (8) in (18), set $\delta_e = \infty$ and obtain

$$(21) \quad U_e(r, \theta, t) = \sum_s \frac{c\theta}{2^{5/2} 3^{1/2} \pi a^{1/2} |\tau_s^\infty|^{1/2} (\sin \theta)^{1/2}} \cdot \frac{1}{(cT)^{3/2}} \exp \left[-\frac{2}{3^{3/2}} \frac{|\tau_s^\infty|^{3/2} \theta^{3/2} a^{1/2}}{(cT)^{1/2}} \right] \cdot \frac{v \left\{ |\tau_s^\infty| 2^{1/3} \left(\frac{\theta(r-a)}{3cT} - 1 \right) \right\}}{v(-|\tau_s^\infty| 2^{1/3})} \cdot \frac{v \left\{ |\tau_s^\infty| 2^{1/3} \left(\frac{\theta(\rho-a)}{3cT} - 1 \right) \right\}}{v(-|\tau_s^\infty| 2^{1/3})}.$$

In the magnetic case we use (9) in (18), set $\delta_m = 0$ and obtain

$$(22) \quad U_m(r, \theta, t) = \sum_s \frac{c\theta |\tau_s^0|^{1/2}}{2^{5/2} 2^{1/3} 3^{1/2} \pi a^{1/2} (\sin \theta)^{1/2}} \cdot \frac{1}{(cT)^{3/2}} \exp \left[-\frac{2}{3^{3/2}} \frac{|\tau_s^0|^{3/2} \theta^{3/2} a^{1/2}}{(cT)^{1/2}} \right] \cdot \frac{v \left\{ |\tau_s^0| 2^{1/3} \left(\frac{\theta(r-a)}{3cT} - 1 \right) \right\}}{v'(-2^{1/3} |\tau_s^0|)} \cdot \frac{v \left\{ |\tau_s^0| 2^{1/3} \left(\frac{\theta(\rho-a)}{3cT} - 1 \right) \right\}}{v'(-2^{1/3} |\tau_s^0|)}.$$

The function v in (21) and (22) is the imaginary part of the Airy function, $v = \text{Im} w$. For large positive z , $v(z)$ is asymptotic to $\frac{1}{2} z^{-1/4} \exp \left[-\frac{2}{3} z^{3/2} \right]$ while for large negative z , $v(z)$ is asymptotic to $(-z)^{-1/4} \sin \left[\frac{2}{3} (-z)^{3/2} + \frac{\pi}{4} \right]$. In the intermediate region the values of $v(z)$ have been tabulated by V. A. Fock [1].

Now let us consider the case of a finitely conducting earth. In the

electric case we will use for γ_s the first two terms of (14) with $\delta = \delta_e$. Upon inserting (8) into (18), with this value of γ_s , and putting $A(k) = 1$, we obtain an expression for $U_e(r, \theta, t)$. For $T < 0$ this expression is zero, as before. For $T > 0$ we again use the saddle point method. Since δ_e is a function of k , the determination of the saddle point is more complicated than in the previous cases. Therefore we neglect the term $-1/(2\gamma_s^\infty \delta_e)$ compared with γ_s^∞ in determining the saddle point. Then we obtain

$$(23) \quad U_e(r, \theta, t) = \sum_s \frac{\propto |\gamma_s^\infty|^{1/2}}{2^{3/2} 3^{1/2} \pi a^{1/2} (\sin \theta)^{1/2}} \cdot \frac{1}{2|\gamma_s^\infty| + \frac{1}{|\gamma_s^\infty| |\delta_e^*|}} \cdot \frac{1}{(cT)^{3/2}} \exp \left[-\frac{2}{3^{3/2}} \frac{\left[|\gamma_s^\infty| + \frac{1}{2|\gamma_s^\infty|} \cdot \frac{1}{|\delta_e^*|} \right]^{3/2} \theta^{3/2} a^{1/2}}{(cT)^{1/2}} \right] \cdot \frac{v \left\{ |\gamma_s^\infty| 2^{1/3} \left(\frac{\theta(r-a)}{3cT} - 1 \right) \right\}}{v(-|\gamma_s^\infty| 2^{1/3})} \cdot \frac{v \left\{ |\gamma_s^\infty| 2^{1/3} \left(\frac{\theta(\rho-a)}{3cT} - 1 \right) \right\}}{v(-|\gamma_s^\infty| 2^{1/3})}.$$

In (23), δ_e^* is the value of δ_e at the stationary point. Its absolute value is given by

$$(24) \quad |\delta_e^*| = \left(\frac{3T}{|\gamma_s^\infty| T_2} \right)^{1/2} \frac{\frac{\epsilon_1}{\epsilon} + \frac{3^{3/2} \sigma_1 \mu_{ca}}{|\gamma_s^\infty|^{3/2}} \left(\frac{T}{T_2} \right)^{3/2}}{\sqrt{\frac{\epsilon_1}{\epsilon} - 1 + \frac{3^{3/2} \sigma_1 \mu_{ca}}{|\gamma_s^\infty|^{3/2}} \left(\frac{T}{T_2} \right)^{3/2}}}.$$

Here $T_2 = \theta a/c$ is the travel time for the pulse to reach the point θ from the source at $\theta = 0$.

In the magnetic case we use the first two terms in (15) for γ_s with $\delta = \delta_m$. Then using (9) in (18) with $A(k) = 1$ we obtain an expression for $U_m(r, \theta, t)$. As before this expression is zero for $T < 0$, while for $T > 0$ we obtain

$$(25) \quad U_m(r, \theta, t) = \sum_s \frac{\propto |\gamma_s^0|^{1/2}}{2^{5/2} 2^{1/3} 3^{1/2} \pi a^{1/2} (\sin \theta)^{1/2}} \cdot \frac{1}{2|\delta_m|^2 (|\gamma_s^0| - |\delta_m^*|) + 1} \cdot \frac{1}{(cT)^{3/2}} \exp \left[-\frac{2}{3^{3/2}} \frac{\left[|\gamma_s^0| - |\delta_m^*| \right]^{3/2} \theta^{3/2} a^{1/2}}{(cT)^{3/2}} \right]$$

$$\cdot \frac{v \left\{ |\chi_s^o| 2^{1/3} \left(\frac{\theta(\rho-a)}{3cT} - 1 \right) \right\}}{v'(-2)^{1/3} |\chi_s^o|} \frac{v \left\{ |\chi_s^o| 2^{1/3} \left(\frac{\theta(r-a)}{3cT} - 1 \right) \right\}}{v'(-2)^{1/3} |\chi_s^o|} \cdot$$

Here δ_m^* is the value of δ_m at the stationary point and $|\delta_m^*|$ is given by

$$(26) \quad |\delta_m^*| = \left(\frac{3cT}{|\chi_s^o| \theta_a} \right)^{1/2} \frac{1}{\sqrt{\frac{\epsilon_1}{\epsilon} - 1 + \frac{\sigma \mu c}{|\chi_s^o|^{3/2}} \frac{(cT)^{3/2}}{a^{1/2} \theta^{3/2}}}} \cdot$$

4. Discussion of results

We have calculated the Hertz vector of the field due to a vertical electric or magnetic dipole with a delta function time dependence. For a perfectly conducting earth we obtain (21) and (22), while for finite conductivity we obtain (23) and (25). Of course (23) reduces to (21) and (25) to (22) as $\sigma_1 \rightarrow \infty$. These results are useful at points beyond the horizon and therefore only the first term ($s = 0$) in each result need be retained. This is so because the other terms have relatively rapidly decaying exponential factors. Consequently our results are rather simple formulas.

Let us examine $U_e(r, \theta, t)$ for the perfectly conducting case when both the source and observation point are on the ground ($\rho = a$ and $r = a$). Then (21) becomes

$$(27) \quad U_e(a, \theta, t) = A_e(\theta) S(T/T_{eo}).$$

Here the amplitude $A_e(\theta)$ is the maximum value of U_e , the build-up time $T_{eo}(\theta)$ is the time at which the maximum occurs and $S(T/T_{eo})$ is the pulse shape factor. A_e , T_{eo} and S are given by

$$(28) \quad A_e = \frac{3^7 c}{|\chi_o^{\infty}|^5 2^{11/2} \pi \epsilon^3 a^2} \sqrt{\frac{1}{\epsilon^7 \sin \theta}} = (1.64 \times 10^{-5}) \frac{1}{\sqrt{\epsilon^7 \sin \theta}} \sim (1.64 \times 10^{-5}) \frac{1}{\theta^4},$$

$$(29) \quad T_{eo} = \frac{4|\chi_o^\infty|^3 a}{3^5 c} \theta^3 = (1.85 \times 10^{-4}) \theta^3,$$

$$(30) \quad S(T/T_{eo}) = (T_{eo}/T)^{3/2} \exp 3[1-(T_{eo}/T)^{1/2}].$$

A graph of the pulse shape factor S is given in Figure 1. U_e is obtained from this graph by multiplying the vertical side by $A_e(\theta)$, which is given by (28).

If the observation point is above the ground ($r > a$), U_e is obtained by multiplying (27) by the time dependent height-gain factor $H_e(T/T_{el}(r))$ defined by

$$(31) \quad H_e\left(\frac{T}{T_{el}(r)}\right) = \frac{v[T_{el}(r)/T - 2^{1/3}|\chi_o^\infty|]}{v[-2^{1/3}|\chi_o^\infty|]}.$$

In (31) the time $T_{el}(r)$ is defined by

$$(32) \quad T_{el}(r) = 2^{1/3}|\chi_o^\infty|\theta(r-a)/3c.$$

A graph of $H_e(T/T_{el}(r))$ is shown in Figure 2. If the source is above the ground ($\rho > a$) a similar height-gain factor must be introduced. Thus in general we have for the perfectly conducting case

$$(33) \quad U_e(r, \theta, t) = A_e(\theta)S(T/T_{eo})H_e(T/T_{el}(r))H_e(T/T_{el}(\rho)).$$

A graph of U_e is shown in Figure 3.

If the times $T_{el}(r)$ and $T_{el}(\rho)$ are both small compared to the build up time T_{eo} , both height-gain factors are effectively equal to unity. Then U_e is essentially the same as it is for both source and receiver on the ground. On the other hand if $T_{el}(r)$ and $T_{el}(\rho)$ are large compared to T_{eo} , the maximum of U_e is less than its value for $r = a$, $\rho = a$ by the factor $H_e(T_o/T_{el}(r))H_e(T_o/T_{el}(\rho))$. In this case the shape of the pulse is also slightly altered.

In case of finite conductivity, (23) shows that U_e is obtained by multiplying (33) by the conductivity factor $C_e(T/T_{eo}, T/T_2)$. This factor is given by

$$(34) \quad C_e = \exp \left[3(T_o/T)^{1/2} \left(1 - \left[1 + \frac{1}{2|\tau_o^\infty|^2 |\delta_e^*|^2} \right]^{3/2} \right) \right] \cdot \frac{1}{1 + \frac{1}{2|\tau_o^\infty|^2 |\delta_e^*|^2} + \frac{1}{2|\tau_o^\infty|^2 |\delta_e^*|^2}}$$

The quantity $|\delta_e^*|$, which depends upon T/T_2 and upon σ_1 , is given by (24) with $s = 0$. Thus we have, denoting by U_e^∞ the result (33) for perfect conductivity,

$$(35) \quad U_e(r, \theta, t) = U_e^\infty(r, \theta, t) C_e(T/T_{eo}, T/T_2).$$

If we insert the expression (33) for U_e^∞ this becomes

$$(36) \quad U_e(r, \theta, t) = A_e(\theta) S(T/T_{eo}) H_e(T/T_{e1}(r)) H_e(T/T_{e1}(\rho)) C_e(T/T_{eo}, T/T_2).$$

A graph of C_e is shown in Figure 4 and one of U_e in Figure 5.

A quite similar analysis is also possible in the magnetic case. From (22) we find that the analogue of (33) for U_m in the case of perfect conductivity is

$$(37) \quad U_m(r, \theta, t) = A_m(\theta) S(T/T_{mo}) H_m(T/T_{m1}(r)) H_m(T/T_{m1}(\rho)).$$

The shape factor S is the same as in the electric case but the amplitude $A_m(\theta)$, the build-up time T_{mo} , the times $T_{m1}(r)$ and $T_{m1}(\rho)$ and the height-gain factors H_m are slightly different. These quantities are given by

$$(38) \quad A_m(\theta) = \frac{3^7 c}{2^{11/2} 2^{1/3} \pi e^3 |\tau_o^\infty|^4 a^2} \cdot \frac{1}{\sqrt{\theta^7 \sin \theta}} = (3.78 \times 10^{-3}) \frac{1}{\sqrt{\theta^7 \sin \theta}} \sim (3.78 \times 10^{-3}) \frac{1}{e^{1/2}},$$

$$(39) \quad T_{mo} = \frac{4 |\tau_o^\infty|^3 a}{3^5 c} \theta^3 = (2.24 \times 10^{-9}) \theta^3,$$

$$(40) \quad T_{m1}(r) = 2^{1/3} |\zeta_o^0| \Theta(r-a)/3c,$$

$$(41) \quad H_m(T/T_{m1}(r)) = \frac{v \left[T_{m1}(r)/T - 2^{1/3} |\zeta_o^0| \right]}{v \left[-2^{1/3} |\zeta_o^0| \right]}.$$

A graph of the height-gain factor H_m is given in Figure 6. This height-gain factor is zero at the ground. Therefore graphs of U_m are given for $r > a$ and $\rho > a$ in Figure 7.

In the case of finite conductivity, we find from (25) that U_m can be obtained by multiplying the result (37) by the conductivity factor $C_m(T/T_{mo}, T/T_2)$. This factor is

$$(42) \quad C_m(T/T_{mo}, T/T_2) = \frac{\exp \left[3 \left(\frac{T_o}{T} \right)^{1/2} \left(1 - \left[1 - \frac{|\delta_m^*|}{|\zeta_o^0|} \right]^{3/2} \right) \right]}{2 |\delta_m^*|^2 (|\zeta_o^0| - |\delta_m^*|) + 1}.$$

The quantity $|\delta_m^*|$ depends upon T/T_2 and upon σ_1 . It is given by (26) with $s = 0$. If we denote the result (37) for perfect conductivity by U_m^∞ we now have

$$(43) \quad U_m(r, \theta, t) = U_m^\infty(r, \theta, t) C_m(T/T_{mo}, T/T_2).$$

If we replace U_m^∞ by means of (37) this becomes

$$(44) \quad U_m(r, \theta, t) = A_m(\theta) S(T/T_{mo}) H_m(T/T_{m1}(r)) H_m(T/T_{m1}(\rho)) C_m(T/T_{mo}, T/T_2).$$

If $\sigma = 4$ mhos/meter, which is the conductivity of sea water, C_m differs from unity by less than one per cent for all values of θ . Therefore in this case U_m is essentially the same as U_m^∞ , which is shown in Figure 7.

The field components can be found from the Hertz vector by equation (1) in the electric case and by equation (21) in the magnetic case. In the electric case $E_\phi = 0$, and $E_\theta = 0$ at $r = a$ in the perfectly conducting case. The main non-zero component of E is E_r . By using (1) and (18) we find, for finite or infinite conductivity, that

$$(45) \quad E_r = -\frac{1}{a} \left[\frac{|\zeta_o^\infty| T_2}{3T} \right]^3 U_\theta.$$

In the magnetic case $E_r = E_\theta = 0$. From (2) and (18) we find for the only non-zero component of E ,

$$(46) \quad E_\phi = \frac{\mu c}{a} \left[\frac{|\mathcal{L}_0^\circ| T_2}{3T} \right]^3 U_m.$$

When the source has a time dependence $f(t)$ instead of $\delta(t)$, the result for the Hertz vector or the field components can be obtained from those for the delta function source. If U^f denotes the Hertz vector (electric or magnetic) due to the source f , and U^δ the corresponding Hertz vector due to the delta-function source, then by superposition

$$(47) \quad U^f(r, \theta, t) = \int_{-\infty}^{\infty} f(\tau) U^\delta(r, \theta, t - \tau) d\tau.$$

From this equation we see that the minimum resolution time in U^f is the appropriate build-up time T_{eo} or T_{mo} . Thus frequencies higher than $1/T_{eo}$ or $1/T_{mo}$ are essentially lost from the pulse. We may describe this by saying that diffraction has the effect of a low pass filter. This is understandable since high frequency fields do not so readily diffract around the earth.

An alternative to (47), which may be easier to compute from, is

$$(48) \quad U^f(r, \theta, t) = \bar{F}(ck^*) U^\delta(r, \theta, t).$$

Here \bar{F} is the Fourier transform of $f(t)$ and k^* is the value of k at the saddle point. It is given by

$$(49) \quad ck^* = \frac{ic}{a} \left[\frac{|\mathcal{L}_0^\circ| T_2}{3T} \right]^{3/2}.$$

Here \mathcal{L}_0° is \mathcal{L}_0^∞ in the electric case and \mathcal{L}_0° in the magnetic case. Although (47) is valid for arbitrary $f(t)$, (48) applies only when $f(t)$ represents a pulse. This limitation results because the phase of \bar{F} was ignored in determining the saddle point.

Finally we must point out a limitation on our result (33) in the electric dipole case for a finitely conducting earth. This limitation pertains to small values of T . For such values of T the stationary value of k or ω is large in the integral (18) for U_e . But our approximate calculation of \mathcal{L}_g , which occurs in

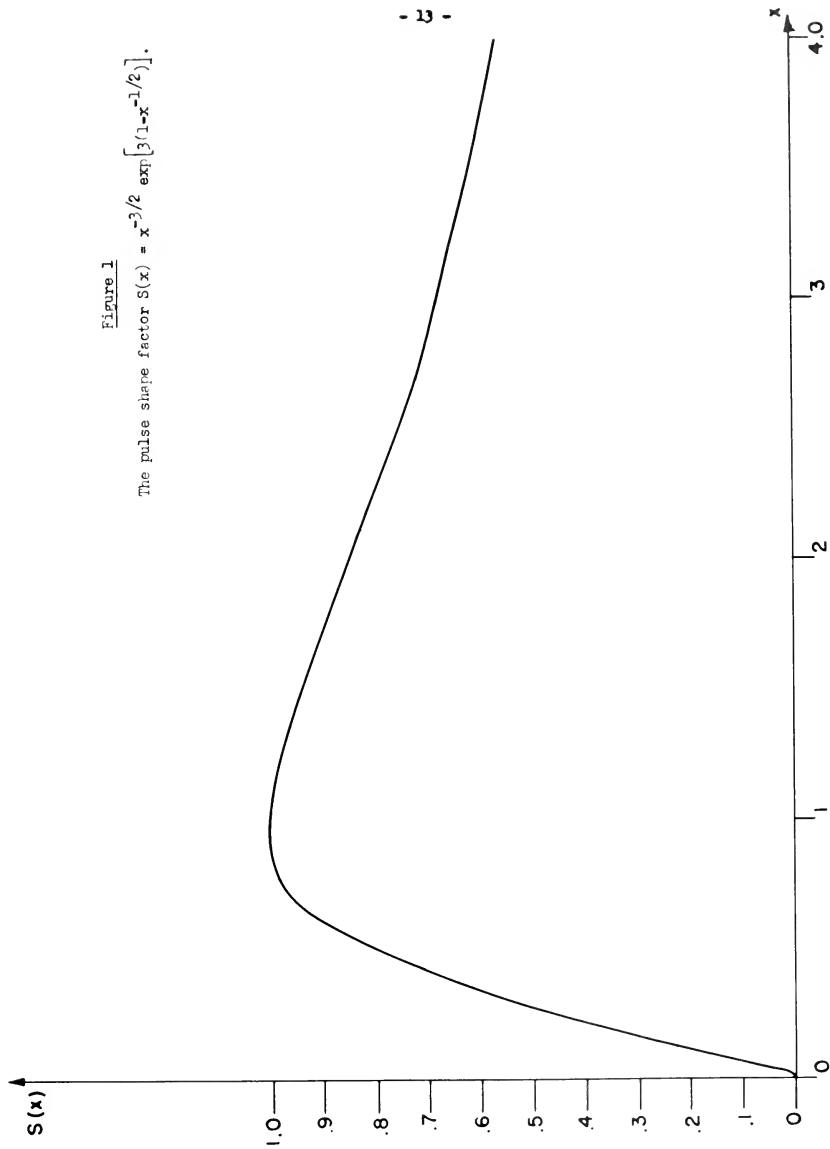
that integral, is based on an expansion for large σ and it is not valid when ω is also large. Therefore the result (23) is not accurate for small T. However this range of T is confined to a small interval which terminates long before the maximum occurs, in the case we have considered. No such limitation occurs in the magnetic dipole case, however, since then the expansion for large σ becomes more accurate when ω is large.

References

- [1] Fock, V.A. - Diffraction of radio waves around the earth's surface; Academy of Sciences, USSR, Moscow, 1946.
- [2] Brenner, H. - Terrestrial Radio Waves, Elsevier, New York, 1949.

Figure 1

The pulse shape factor $S(x) = x^{-3/2} \exp[3(1-x^{-1/2})]$.



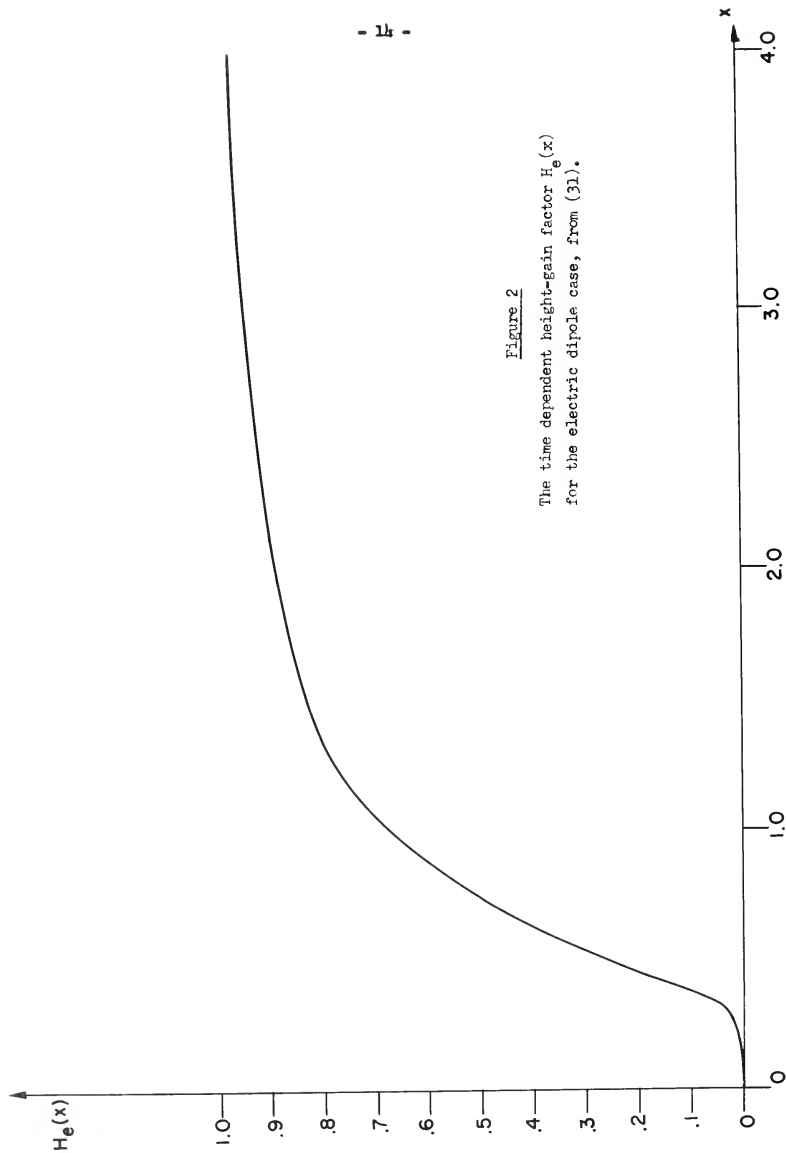


Figure 2
The time dependent height-gain factor $H_e(x)$
for the electric dipole case, from (31).

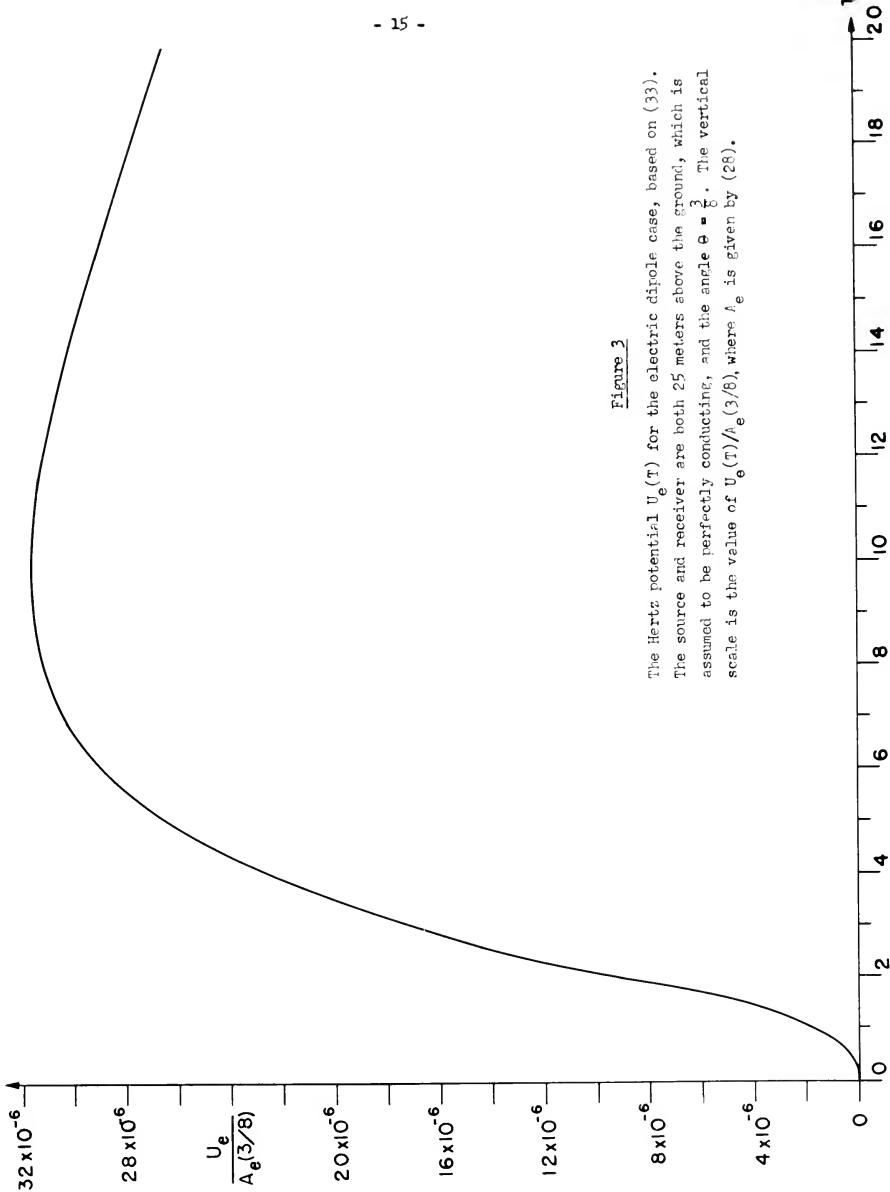


Figure 3

The Hertz potential $U_e(r)$ for the electric dipole case, based on (33). The source and receiver are both 25 meters above the ground, which is assumed to be perfectly conducting, and the angle $\theta = \frac{3}{8}$. The vertical scale is the value of $U_e(r)/A_e (3/8)$, where A_e is given by (25).

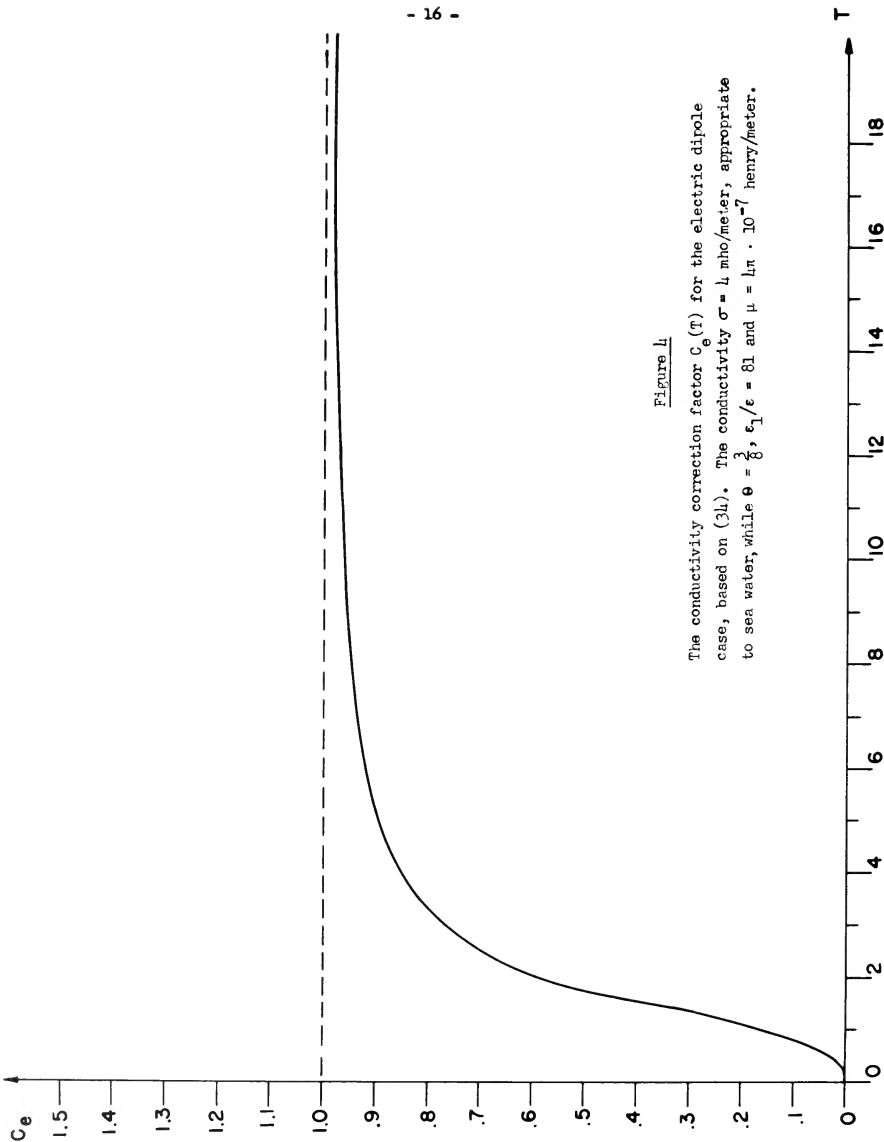


Figure 4

The conductivity correction factor $C_e(T)$ for the electric dipole case, based on (34). The conductivity $\sigma = 4 \text{ mho/meter}$, appropriate to sea water, while $\theta = \frac{3}{8}$, $\epsilon_1/\epsilon = 81$ and $\mu = 4\pi \cdot 10^{-7} \text{ henry/meter}$.

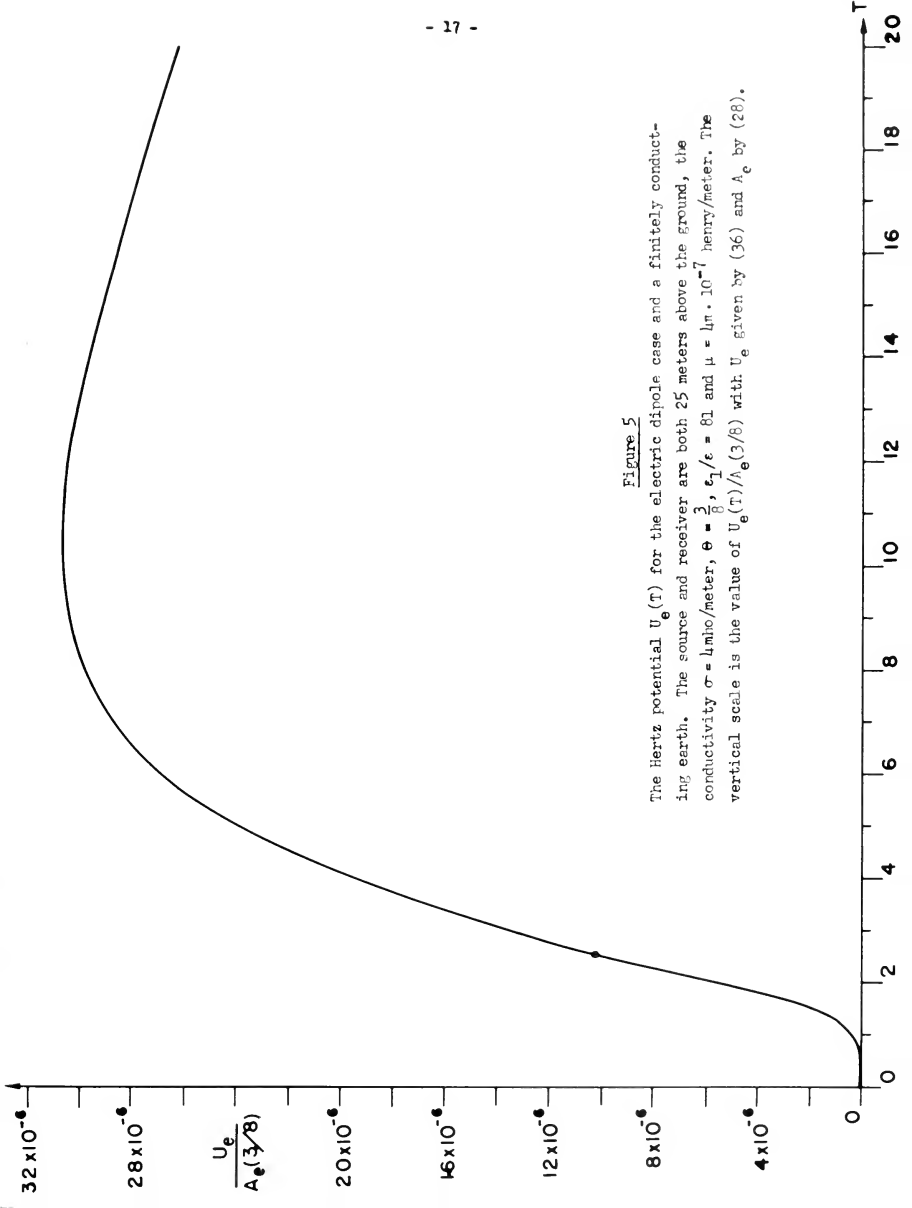


Figure 5

The Hertz potential $U_e(T)$ for the electric dipole case and a finitely conducting earth. The source and receiver are both 25 meters above the ground, the conductivity $\sigma = 4 \text{ mho/meter}$, $\theta = \frac{\pi}{8}$, $\epsilon_1/\epsilon = 81$ and $\mu = 4\pi \cdot 10^{-7}$ henry/meter. The vertical scale is the value of $U_e(T)/A_e(3/8)$ with U_e given by (36) and A_e by (28).

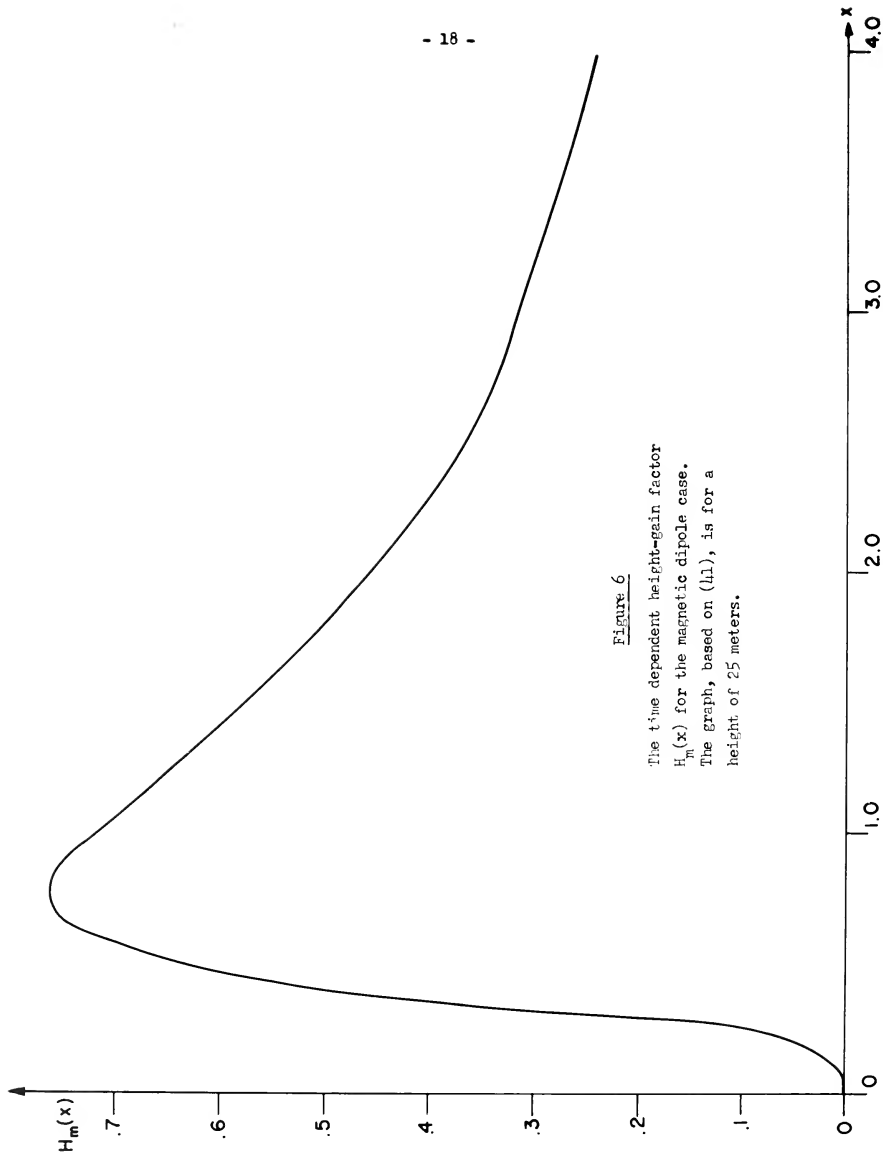
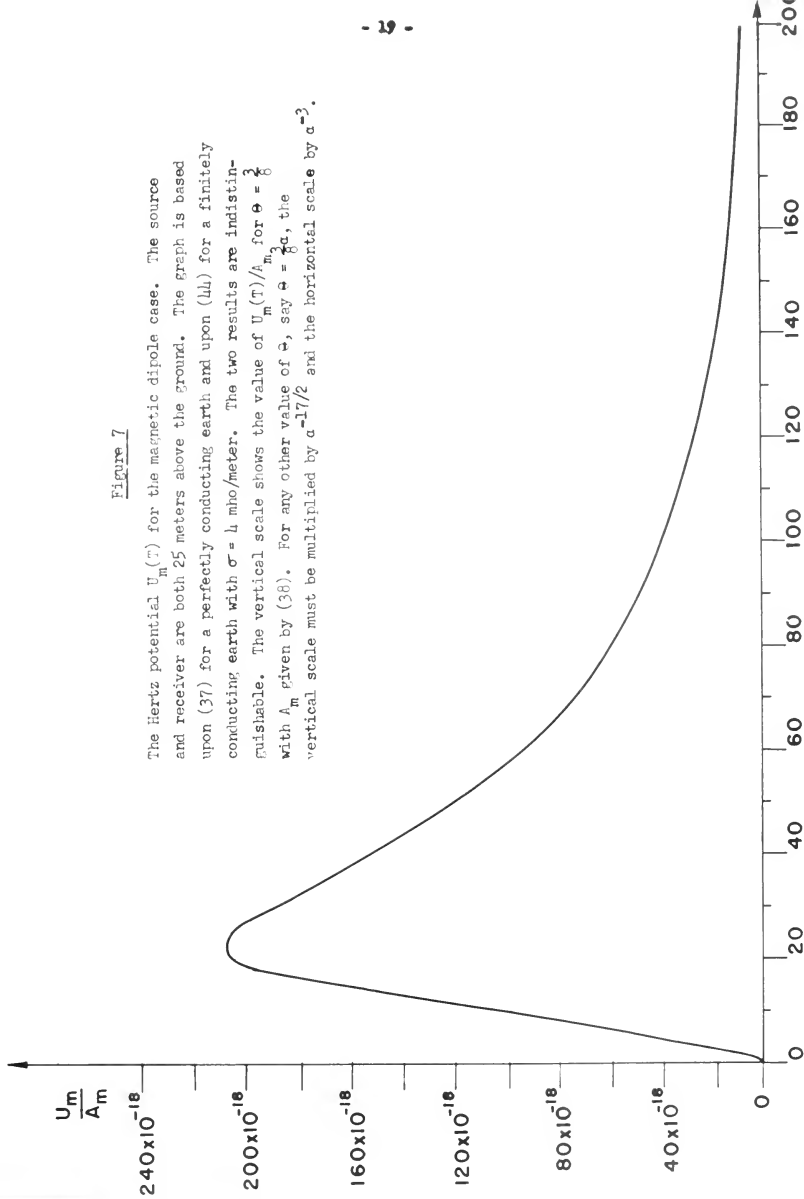


Figure 6

The time dependent height-gain factor $H_m(x)$ for the magnetic dipole case. The graph, based on (11), is for a height of 25 meters.

Figure 7

The Hertz potential $U_m(r)$ for the magnetic dipole case. The source and receiver are both 25 meters above the ground. The graph is based upon (37) for a perfectly conducting earth and upon (44) for a finitely conducting earth with $\sigma = 4$ mho/meter. The two results are indistinguishable. The vertical scale shows the value of $U_m(r)/A_m$ for $\theta = \frac{3}{8}$ with A_m given by (38). For any other value of θ , say $\theta = \frac{3}{8}\alpha$, the vertical scale must be multiplied by $\alpha^{-17/2}$ and the horizontal scale by α^{-3} .



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Levy

Propagation of electro
magnetic pulses around
the earth.

$$P = \int_0^1 dx \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \phi'^2 + V(\phi) \right) = P_{\text{eff}}(g, \lambda, \mu)$$

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Levy

Propagation of electromagnetic pulses around the earth.

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